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Performance Analysis of Multiuser Maximal Ratio Combining Receiver System over Generalized-*k* Fading Channels with Transmit Antenna Selection

Chandana Brahma
Department of Electronics and Communication Engineering
North-Eastern Hill University
Shillong, India
E-mail: donabrahma94@gmail.com

Rajkishur Mudoi
Department of Electronics and Communication Engineering
North-Eastern Hill University
Shillong, India
E-mail: rajkishur@gmail.com

Abstract: This paper presents the different expressions of performance measure for multi-user and multiple-input multiple-output (MIMO) system over Generalized- k fading channels. Transmit antenna selection at the base station has been considered for downlink transmission, and maximal ratio combining (MRC) is performed at the receiver to improve the performance of the system. Outage probability, average bit error rate (ABER) performance of BPSK modulation scheme and channel capacity are also derived and plotted for analysis. The effect of multiple fading parameters has been observed both mathematically and analytically.

Keywords: Generalized- k distribution, multiple-input multiple-output (MIMO), transmit antenna selection with maximal ratio combining (TAS/MRC), ABER, outage probability, channel capacity.

I. INTRODUCTION

In a wireless communication, the problems of signal degradation because of channel fading can be mitigated with the help of various technologies that have been developed for both the transmitter as well as for the receiver. One such technology is the MIMO is antenna systems that consists large number of transmit antennas and receives antennas. MIMO provides improvements in channel robustness as well as in channel throughput. The MIMO system provides the best diversity gain of the signals received at the receiving end also it has got the ability to handle the external noise effect in the most appropriate manner than other antenna systems as to reconstruct the same signal at the receiving end [1][2]. However, in MIMO schemes, for a large number of antennas, the hardware complexity and the price of the system goes high. Continuous transmissions from multiple antennas also cause inter-antenna interference, the need of synchronization etc. Transmit antenna selection with maximal ratio combining at the receiver (TAS/MRC) is one of the most popular MIMO system with antenna selection to overcome these disadvantages [3]. In telecommunication, MRC is a method of diversity combining in which the signals from all the channels are co-phased and proportionally weighted and are combined together to maximize the output SNR. In TAS/MRC scheme, the channel state information (CSI) of all link has been fed back to the transmitter and based on CSI the transmitter allots the best antenna for each user. Each user of the system performs MRC to improve the receive signal quality. TAS/MRC scheme has been investigated over various flat fading channels in the past [3] [4] [5]. In [6] the expression for the ABER for TAS/MRC systems under Hoyt fading channels has been presented and in [7] the expressions for both outage probability and exact bit error rate for TAS/MRC scheme has been shown. These encourages to know the performance of the TAS / MRC system over Generalized- k (K_G) fading channels. K_G Fading is a composite fading that consist of Nakagami and Gamma distribution. In [8] it has been mentioned that the main advantage of employing the K_G distribution is that it makes mathematical performance analysis much easier to handle as compared to Lognormal-based models such as the Nakagami- m or the R-L model. The K_G distribution is general enough to model the fading and shadowing phenomena encountered in mobile communication channels. In [9], the outage probability and the channel capacity over K_G is analyzed and evaluated. However, a detailed performance analysis for the performance of more general receiver structures such as TAS/MRC operating over K_G channel is not available and thus is the topic of our contribution.

The next sections of the paper are organized as follows. The system model is described in section II. The outage probability, ABER and channel capacity analysis are presented in sections III, IV and V, respectively. In section VI, numerical results and discussions are presented. Concluding remarks are presented in section VII.

II. SYSTEM MODEL

We have considered a multiuser, MIMO wireless communication and TAS/MRC system as shown in Figure 1. The total number of users are denoted by R . The base station has N_t transmit antennas and each user has N_r receive antennas. The base station is selecting single transmitter at a time. The fading path experienced by the channel between the base station and a receiver is assumed to be slow and flat fading channel.

For a K_G fading channel, the equivalent baseband received signal can be expressed as [9]

$z = sX + n,$ (1)

where s is the transmitted symbol that can take values of different modulation alphabets such as M-phase shift keying (PSK) and M-quadrature amplitude modulation (QAM), and n is the additive white Gaussian noise (AWGN). The instantaneous SNR per received symbol is

$\gamma = \frac{X^2 E_s}{N_0}$. Where $E_s = E\left[|s|^2\right]$ with $|.|$ and $E[.]$ denoting absolute value and expectation operator respectively. N_0 is the single-sided power spectral density of the AWGN. The corresponding average SNR is $\bar{\gamma} = \frac{\Omega k E_s}{N_0}$.

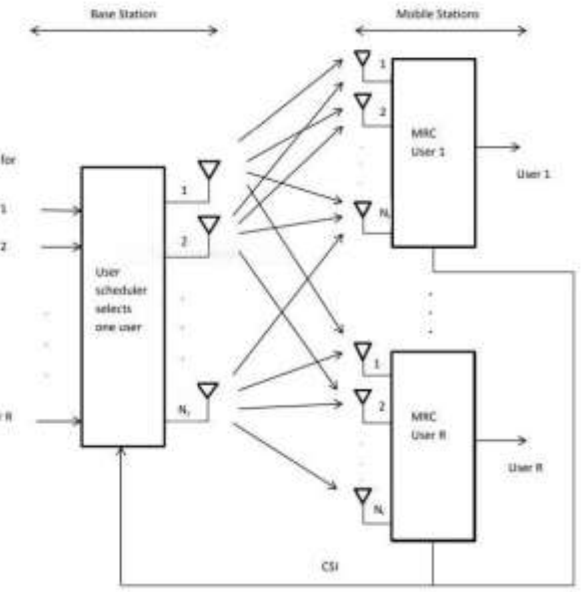

Figure 1: Block diagram of the TAS/MRC system.

The fading amplitude X is a random variable following the K_G distribution. Then the probability density function (PDF) of X is given by [9]

$$f_X(x) = \frac{4m^{\left(\frac{\beta+1}{2}\right)} x^{\beta}}{\Gamma(m)\Gamma(k)\Omega^{\left(\frac{\beta+1}{2}\right)}} K_{\alpha}\left[2\left(\frac{m}{\Omega}\right)^{\frac{1}{2}} x\right]; x \geq 0, \quad (2)$$

with $\alpha = k - m$, $\beta = k + m - 1$, where k, m are the distribution shaping parameters, $K_{\alpha}(.)$ is the modified Bessel function of order α [10, (8.407.1)], $\Gamma(.)$ is the Gamma function [10, (8.310.1)]. The mean power defined as $\Omega \triangleq \frac{E\left[X^2\right]}{k}$. Since K_G is a two parameter distribution, (2) can describe various fading and shadowing models by using different value combinations for k and/or m .For $k \to \infty$, it can approximate the Nakagami-m distribution and for $m = 1$, it concurs with the k distribution and approximately models R-L fading conditions, while for $m \to \infty$ and $k \to \infty$, (2) approaches the AWGN channel [9].

It was testified and verified that both a single K_G RV and the sum of independent K_G RVs can be closely approximated by a single Gamma RV [11]. Denoting γ_i as a Gamma distributed RV with a shape parameter ρ and a scale parameter θ, the PDF of the instantaneous SNR γ_i is given as [12][13]

$$f_{\gamma_i}(\gamma) = \frac{\theta^{-\rho}}{\Gamma(\rho)} \gamma^{\rho-1} \exp\left(-\frac{\gamma}{\theta}\right). \quad (3)$$

Where $\theta = (AF - \varepsilon)\bar{\gamma}$, $\rho = \frac{N_r}{AF - \varepsilon}$ and $AF = \frac{1}{m} + \frac{1}{k} + \frac{1}{mk}$.

Here θ is the scale parameter, AF is the amount of fading, and ε is the adjustment parameter. m and k are the fading parameters. Therefore the cumulative distribution function (CDF) of the instantaneous SNR over a K_G fading channel is given as

$$F_{\gamma_i}(\gamma) = \frac{\theta^{-\rho}}{\Gamma(\rho)} \int_0^{\gamma} \gamma^{\rho-1} \exp\left(-\frac{\gamma}{\theta}\right) d\gamma. \quad (4)$$

Simplifying using [10, (3.381.1)],

$$F_{\gamma_i}(\gamma) = \left[\frac{1}{\Gamma(\rho)} g\left(\rho, \frac{\gamma}{\theta}\right)\right], \quad (5)$$

where, $g\left(\rho, \frac{\gamma}{\theta}\right) = \int_0^{\frac{\gamma}{\theta}} \exp(-t) t^{\rho-1} dt$ is the lower incomplete Gamma function [10].

In TAS/MRC system, the transmitting antenna corresponding to the highest channel gain is selected when MRC diversity technique is performing at the receiver. Then the CDF of post processing SNR in such a system can be expressed as

$$F_{\gamma}(\gamma) = \left[\frac{1}{\Gamma(\rho)} g\left(\rho, \frac{\gamma}{\theta}\right)\right]^{N_t R}. \quad (6)$$

Then the PDF of output SNR in the multiuser, TAS/MRC system over K_G fading channel is

$$f_{\gamma}(\gamma) = \frac{N_t R}{\left[\Gamma(\rho)\right]^{N_t R}} \left[g\left(\rho, \frac{\gamma}{\theta}\right)\right]^{N_t R - 1} \frac{d}{d\gamma}\left[g\left(\rho, \frac{\gamma}{\theta}\right)\right]. \quad (7)$$

Differentiating the incomplete Gamma function using [14, (6.5.25)],

$$f_{\gamma}(\gamma) = \frac{N_t R}{\theta^{\rho}\left[\Gamma(\rho)\right]^{N_t R}} \left[g\left(\rho, \frac{\gamma}{\theta}\right)\right]^{N_t R - 1} \gamma^{\rho-1} e^{-\frac{\gamma}{\theta}}. \quad (8)$$

The gamma function can be written in infinite series using [15, (1.7)], then the PDF of post processing SNR is obtained as

$$f_\gamma(\gamma)=\frac{N_tR}{\left[\Gamma(\rho)\right]^{N_tR}}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\cdots\sum_{n_{N_tR-1}=0}^{\infty}\frac{\left(\frac{1}{\theta}\right)^{N_tR\rho+\sum_{i=1}^{N_tR-1}n_i}}{\prod_{i=0}^{N_tR=0}(\rho)_{n_i+1}}$$
$$\times\left(e^{-N_tR\frac{\gamma}{\theta}}\right)\gamma^{\left(N_tR\rho+\sum_{i=0}^{N_tR-1}n_i\right)-1}. \quad (9)$$

III. OUTAGE PROBABILITY

Outage Probability is a standard performance criterion characteristic of diversity systems operating over fading channels. It is denoted by P_{out} and is defined as the probability that the instantaneous error probability exceeds a specified value or equivalently, the probability that the output SNR, γ, falls below a certain specified threshold, γ_{th} [1]. It can be given mathematically as [9]

$$P_{out}(\gamma_{th})=F_\gamma(\gamma_{th}). \quad (10)$$

For the TAS/MRC system an expression for the outage probability can be obtained from (6) by putting $\gamma=\gamma_{th}$ as

$$P_{out}(\gamma_{th})=\left[\frac{1}{\left[\Gamma(\rho)\right]}g\left(\rho,\frac{\gamma_{th}}{\theta}\right)\right]^{N_tR}. \quad (11)$$

Defining, $\bar{\gamma}_N \triangleq \frac{\bar{\gamma}}{\gamma_{th}}$ as the normalized average branch SNR, the expression of outage probability can be given as

$$P_{out}=\left[\frac{1}{\left[\Gamma(\rho)\right]}g\left(\rho,\frac{1}{(AF-\varepsilon)\bar{\gamma}_N}\right)\right]^{N_tR}. \quad (12)$$

IV. AVERAGE BIT ERROR RATE

The error rate per bit (BER) is dependent on the fading distribution and modulation technique. The ABER can be calculated averaging the conditional error probability (CEP), i.e., the error rate under AWGN, over the output SNR, which is given as [6]

$$\bar{P}_e=-\int_0^\infty P_e'(\gamma)F_\gamma(\gamma)d\gamma. \quad (13)$$

Where, $P_e'(\gamma)=-\frac{\zeta^\eta\gamma^{\eta-1}e^{-\zeta\gamma}}{2\Gamma(\eta)}$ is the first order derivative of CEP and $\Gamma(\eta)$ is the Gamma function and for BPSK modulation, the value of the constants $\zeta=1$ and $\eta=0.5$ are [6]. Putting the value of $P_e'(\gamma)$ and $F_\gamma(\gamma)$ from (6), ABER can be given as

$$\bar{P}_e=\frac{\zeta^\eta}{2\Gamma(\eta)\left[\Gamma(\rho)\right]^{N_tR}}\int_0^\infty\gamma^{\eta-1}e^{-\zeta\gamma}\left[g\left(\rho,\frac{\gamma}{\theta}\right)\right]^{N_tR}d\gamma \quad (14)$$

Using [15, (1.7)] in (14) and simplifying, the ABER can be obtained as

$$\bar{P}_e=\frac{\zeta^\eta}{2\Gamma(\eta)\left[\Gamma(\rho)\right]^{N_tR}}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\cdots\sum_{n_{N_tR}=0}^{\infty}\frac{\left(\frac{1}{\theta}\right)^{\left(N_tR\rho+\sum_{i=1}^{N_tR}n_i\right)}}{\prod_{i=1}^{N_tR}(\rho)_{n_i+1}}$$
$$\times\int_0^\infty\gamma^{\left(N_tR\rho+\sum_{i=1}^{N_tR}n_i+\eta\right)-1}e^{-\gamma\left(\zeta+\frac{N_tR}{\theta}\right)}d\gamma. \quad (15)$$

The integration in (15) can be solved using [10, (3.381.4)] and after simplification, the expression of ABER can be given as

$$\bar{P}_e=\frac{\zeta^\eta}{2\Gamma(\eta)\left[\Gamma(\rho)\right]^{N_tR}}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\cdots\sum_{n_{N_tR}=0}^{\infty}\frac{\left(\frac{1}{\theta}\right)^{\left(N_tR\rho+\sum_{i=1}^{N_tR}n_i\right)}}{\prod_{i=1}^{N_tR}(\rho)_{n_i+1}}$$
$$\times\frac{1}{\left(\zeta+\frac{N_tR}{\theta}\right)^{\left(N_tR\rho+\sum_{i=1}^{N_tR}n_i+\eta\right)}}\Gamma\left(N_tR\rho+\sum_{i=1}^{N_tR}n_i+\eta\right). \quad (16)$$

V. CHANNEL CAPACITY

Ergodic capacity is the maximum data rate that can be sent over the channel at an arbitrarily small BER, without any complexity constraints. The ergodic capacity of a fading channel with receiver CSI can be obtained as [16]

$$C_{erg}=B\int_0^\infty\log_2(1+\gamma)f_\gamma(\gamma)d\gamma. \quad (17)$$

Where B is the channel bandwidth, and $f_\gamma(\gamma)$ is the PDF of the output SNR. Putting the expression of $f_\gamma(\gamma)$ from (9) into (17), the expression can be obtained as[17]

$$C_{erg}=\frac{BN_tR}{\ln(2)\left[\Gamma(\rho)\right]^{N_tR}}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\cdots\sum_{n_{N_tR-1}=0}^{\infty}\frac{\left(\frac{1}{\theta}\right)^{N_tR\rho+\sum_{i=1}^{N_tR-1}n_i}}{\prod_{i=1}^{N_tR-1}(\rho)_{n_i+1}}$$
$$\times\int_0^\infty\ln(1+\gamma)\left(e^{-N_tR\frac{\gamma}{\theta}}\right)\gamma^{\left(N_tR\rho+\sum_{i=0}^{N_tR-1}n_i\right)-1}d\gamma. \quad (18)$$

Solving the integral, the ergodic capacity of the channel can be given as

$$C_{erg} = \frac{BN_tR}{\ln(2)\left[\Gamma(\rho)\right]^{N_tR}} \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\cdots\sum_{n_{N_tR-1}=0}^{\infty} \frac{\left(\frac{1}{\theta}\right)^{N_tR\rho+\sum_{i=1}^{N_tR-1} n_i}}{\prod_{i=1}^{N_tR-1}(\rho)_{n_i+1}}$$

$$\times I_{\left(N_tR\rho+\sum_{i=1}^{N_tR-1} n_i\right)}\left(\frac{N_tR}{\theta}\right). \quad (19)$$

Where, $I_n(\mu)=\int_0^{\infty} t^{n-1}\ln(1+t)e^{-\mu t}dt$ and for integer n, $I_n(\mu)$ can be given as [18], $I_n(\mu)=(n-1)!e^{\mu}\sum_{k=1}^{n}\frac{\Gamma(-n+k,\mu)}{\mu^k}$.

VI. NUMERICAL RESULTS AND DISCUSSION

In this section, the mathematical expressions of outage probability, ABER and the channel capacity have been numerically evaluated and plotted for demonstration. The outage probability vs. normalized SNR per branch ($\bar{\gamma}_N$) has been plotted for different number of users in Figure 2. It is seen that the outage performance improves with increase in the number of users for a fixed value of transmit and/or receive antennas and fading parameters. In Figure 3, ABER vs. average SNR per branch ($\bar{\gamma}$) has been plotted for BPSK modulation scheme considering different number of users. It is observed that the ABER decrease as the number of users increases. In this Figure 3, $N_t=3$, $N_r=3$, $m=2$ and $k=1$. Ergodic capacity vs. average SNR per branch ($\bar{\gamma}$) has been plotted in Figure 4 for different N_r. The capacity increases as the number of receive antennas per user N_r increases, for a fixed value of transmit antennas and fading parameters. This is because the large number of N_r provides more selection possibilities which brings large post processing SNR. In Figure 5, ergodic capacity vs. average SNR per branch ($\bar{\gamma}$) has been plotted for different fading parameters. The capacity degrades as the fading parameter m increases.

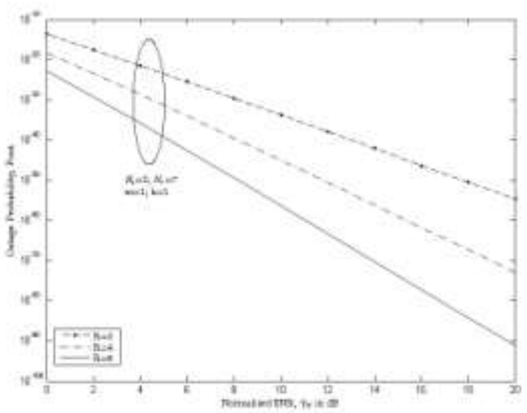

Figure 2. Outage probability vs. normalized SNR per branch ($\bar{\gamma}_N$) for different R.

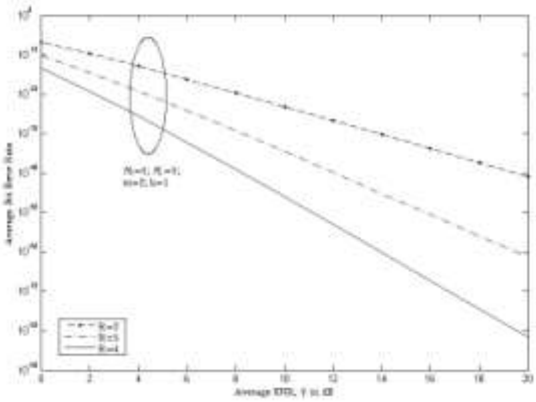

Figure 3. ABER vs. average SNR per branch ($\bar{\gamma}$) for BPSK modulation.

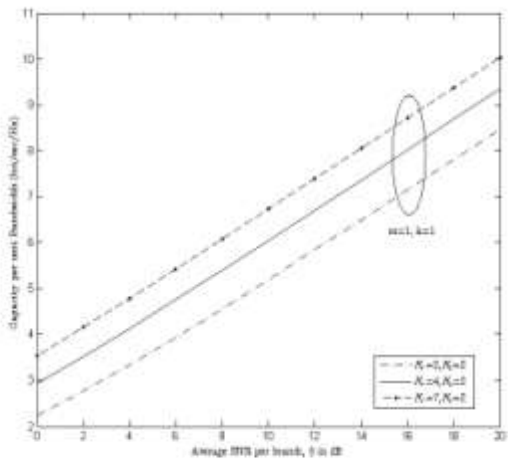

Figure 4. Ergodic capacity vs. average SNR per branch ($\bar{\gamma}$) with $R=2$.

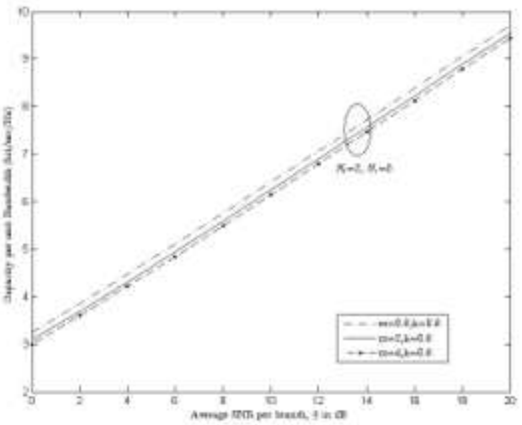

Figure 5. Ergodic capacity vs. average SNR per branch ($\bar{\gamma}$) for different fading parameters and $R = 2$.

VII. CONCLUSION

In the above work, the expressions for PDF and CDF of multiuser TAS/MRC system have been derived over generalized- k fading channels. Also, the expressions of outage probability and ABER have been derived in terms of incomplete Gamma function. The closed form expressions of the channel capacity is presented. The effect of fading parameters and the number of transmit and/or receive antennas on the system performance are analyzed.

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AUTHOR'S BIOGRAPHIES

Chandana Brahma received her B.Tech degree in Electronics and Communication Engineering from Central Institute of Technology, Kokrajhar in Assam, India. She is currently pursuing her M. Tech degree in Electronics and Communication Engineering in North Eastern Hill University, Shillong in Meghalaya, India. Her research interest is in the field of wireless communication.

Rajkishur Mudoi received M. Tech degree from Department of Electronics, Tezpur University, Assam, India and Ph. D. degree from Electronics and Communication Engineering Department, NEHU, Shillong, India. Presently, he is Assistant Professor in Department of Electronics and Communication Engineering, North Eastern Hill University, Shillong, Meghalaya, India. His research interests include wireless communication and VLSI design.